The locked fraction of vacuum entanglement: an exact pure-cut value, a cutoff-pinned transition, and the multipartite routing of entanglement transfer

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Abstract

We study how much of the correlation that the free-field vacuum maintains between spatial regions is *locked* — present in the mutual information but carrying no distillable entanglement — and how entanglement moves through a vacuum when it is transferred between sites. Three results. (i) *An exact value:* for any region of any pure global state, the distillable entanglement across the cut equals the entanglement entropy, so exactly one half of the mutual-information account $I(A:\bar{A}) = 2S(A)$ is locked; the region-versus-complement locked fraction is 1/2 identically. (ii) *A cutoff-pinned transition:* for two balls of radius R at centre separation $d = \kappa R$ in the massless 3+1D lattice scalar vacuum, the locked fraction $f = 1 - E_N/I$ rises from its cliff at the negativity-vanishing edge to 1; the contour $f = 0.68$ sits at $\kappa^*(R) = 2 + g/R$ with $g = 1.83 \pm 0.02$ lattice units, *independent of* R across $R = 3$–8 (the $R = 8$ value was predicted to 0.04% before being computed). The transition between the universal pure-cut value 1/2 and full locking is therefore a cutoff-scale structure: in the continuum limit at fixed shape it collapses onto contact. (iii) *Multipartite routing:* re-auditing a beam-splitter entanglement-transfer protocol in a monogamy-respecting currency (squared logarithmic negativity), we find no pairwise path between accounts: the source pairwise account collapses before the destination account forms, the budget passes through a configuration that is 100% genuinely multipartite, and the books close exactly at settlement. We also quantify how strongly pairwise accounts under-represent a single site's budget in the vacuum itself (CKW saturation 11% at criticality, 40% in a gapped chain). All results use exact Gaussian-state methods; every figure regenerates from short public scripts.

1 Introduction and summary

The entanglement structure of free-field vacua is by now classical territory: entropies and mutual informations of single and multiple regions are known analytically in many cases [1], and the logarithmic negativity between *separated* regions is known to vanish abruptly beyond a separation comparable to the region size — the "negativity sphere" of Klco and Savage [2, 3]. Beyond that edge the regions remain correlated (the mutual information decays only as a power law in the massless case) while no entanglement can be distilled from them by local operations: the correlation is, in the bookkeeping sense we adopt here, *locked*.

*Contact via emergent-gravity.com. Numerical work performed in human–AI collaboration (Claude, Anthropic); all results are reproducible from the public scripts cited in Sec. 7.

This paper treats the locked/liquid decomposition of vacuum correlation as an object of study in its own right and reports three results about it, one exact and elementary, one numerical with a predictive test, and one dynamical.

The locked fraction. For two disjoint regions A, B of the vacuum define

$$f(A,B) \;=\; 1 - \frac{E_N(A:B)}{I(A:B)}, \tag{1}$$

with I the mutual information and E_N the logarithmic negativity. Since E_N upper-bounds distillable entanglement [4, 5], f *underestimates* the truly locked share; where $E_N = 0$ exactly (PPT), nothing is distillable and $f = 1$ is exact. (E_N and I obey no general ordering; f is meaningful in the regime $E_N < I$, which for separated vacuum regions is everywhere beyond the immediate neighbourhood of contact.)

Result 1 (exact, Sec. 3). For a region against its full complement in any *pure* global state — the "observable patch" configuration — the distillable entanglement equals $S(A)$ exactly, while the account is $I(A : \bar{A}) = 2S(A)$. The locked fraction of any pure cut is therefore exactly 1/2, independent of the region, the state, and the theory. (The E_N-based proxy (1) is *not* valid on a pure cut: there $E_N = S_{1/2}(A) > S(A)$. The exact statement uses distillable entanglement, for which pure-state distillation is a theorem.)

Result 2 (numerical, Sec. 4). For two balls of radius R at centre distance $d = \kappa R$ in the massless scalar lattice vacuum in three spatial dimensions, the locked fraction (1) rises from a cliff located at the negativity-vanishing edge to 1. Tracking the contour $f = 0.68$ (a value chosen for an external reason recorded in Sec. 6; the structure is the same for any contour in the transition) across five ball sizes gives

$$\kappa^*(R) \;=\; 2 \;+\; \frac{g}{R}, \qquad g = 1.83 \pm 0.02 \;\;\text{(lattice units)}, \tag{2}$$

i.e. the contour sits at a *fixed absolute gap* between the ball surfaces, not at a fixed shape ratio. The law was used to predict the $R = 8$ contour ($\kappa^* = 2.229$) before computing it (2.228). The scheme-independent content is structural: the entire transition between the universal 1/2 and full locking is pinned to the cutoff scale and collapses onto contact ($\kappa \to 2$) in the continuum limit at fixed shape. The number 1.83 itself is regularization-dependent furniture.

Result 3 (dynamical, Sec. 5). Monogamy makes squared negativity (the contangle of Adesso and Illuminati [9, 10]) the natural conserved-style currency for Gaussian states: pairwise accounts plus a CKW-type remainder never exceed the budget [7, 8]. In the vacuum itself the pairwise sector is small: summing the contangle of one site with all site-pairs exhausts only 11.3% of its budget in a near-critical chain (39.9% gapped); the remainder is genuinely multipartite. (By contrast the same decomposition attempted with mutual information *over*-counts the budget by up to a factor 27 — classical redundancy, not a conservation currency.) Tracking a beam-splitter protocol that imports an external two-mode-squeezed pair into two distant chain sites, the contangle portfolio of one site shows: the site's vacuum account collapses almost immediately; the destination account remains exactly zero until the protocol is half done; in between, at coupling $\tau \simeq 0.31$, the budget is 100% multipartite; and at settlement the books close exactly ($\tau(A{:}B) = 9.0000$ against budget 9.0000, collective 0.0000). Within this protocol there is no pairwise path between accounts: *all entanglement transfer clears through the genuinely multipartite sector.*

2 Methods

All states are Gaussian; all quantities are exact functionals of covariance matrices [11, 10].

Vacua. (a) 1+1D: harmonic chain of $N = 240$–400 sites, periodic, $K_{ii} = m^2 + 2$, $K_{i,i\pm1} = -1$, with $m = 10^{-3}$ (near-critical) and $m = 0.1$ (gapped). (b) 3+1D: cubic lattice L^3, periodic, massless ($k = 0$ mode excluded; compact boson), with L up to 64. For the 3D vacuum we exploit translation invariance: the position-space covariances $X = \frac{1}{2}K^{-1/2}$, $P = \frac{1}{2}K^{1/2}$ are obtained by FFT of $1/2\omega_k$ and $\omega_k/2$, reducing region calculations to submatrix extraction — no diagonalization of the full lattice. This removes the usual $L^3 \times L^3$ ceiling and is the enabling trick for the $R = 8$, $L = 64$ runs.

Measures. Entropies from symplectic spectra; for numerical stability all spectra are computed from the symmetrized form $\mathrm{eig}(X^{1/2}MX^{1/2})$ — the unsymmetrized product $\mathrm{eig}(XM)$ produced a *negative mutual information* at $L = 32$ (catastrophic cancellation), an impossible value that we document because it is a trap others will meet. Logarithmic negativity from the partial transpose in covariance form (p-sign flip on one party). Rényi-1/2 entropy for pure-cut negativity, $E_N(A : \bar{A}) = S_{1/2}(A)$. Contangle proxy $\tau = E_N^2$ [9].

Caveats stated once. Periodic images (separations are kept $\leq L/3$); a single scalar species; E_N as distillability proxy (conservative for f, see above); interpolated contour crossings span a steep cliff (the five-size consistency of (2) at the 10^{-3} level is itself the evidence that the interpolation is controlled).

3 Result 1: the pure cut is half locked, exactly

Proposition 1. *Let $\rho_{A\bar{A}} = |\Psi\rangle\langle\Psi|$ be pure. Then the distillable entanglement across the cut is $E_D(A : \bar{A}) = S(A)$, the mutual information is $I(A : \bar{A}) = 2S(A)$, and the locked fraction $1 - E_D/I = 1/2$ for every region A and every pure $|\Psi\rangle$.*

Proof. $E_D = S(A)$ for pure states is the standard distillation/dilution theorem [6]; $I = S(A) + S(\bar{A}) - S(A\bar{A}) = 2S(A)$ since $S(\bar{A}) = S(A)$ and $S(A\bar{A}) = 0$. □

Trivial as it is, this fixes a universal boundary value for the locked landscape: the most inclusive cut a local observer can make — a patch against everything beyond — is exactly half locked, in any theory, at any scale. The interest of the proposition is what it forces on the two-region problem: any locked fraction strictly between 1/2 and 1 must come from geometry, and Sec. 4 locates where.

We note also that the E_N-version of the pure-cut fraction is *negative* (since $S_{1/2} > S_1$): on pure cuts negativity exceeds the account, a reminder that (1) is a separated-region tool.

4 Result 2: the locked transition is pinned to the cutoff

Two balls of radius R (sites within Euclidean distance R of the centre, torus metric), centres separated by d along a lattice axis, massless 3D vacuum, $L = 64$.

The curve. $f(\kappa{=}d/R)$ has three regimes: (i) near contact, E_N is large and f is not meaningful; (ii) a cliff at $\kappa \approx 2.1$–2.3 whose position is the negativity-vanishing edge of Refs. [2, 3] seen from the locked side; (iii) a smooth rise to $f = 1$ (complete locking) beyond it. Representative values at $R = 8$: $f(2.125) = 0.09$, $f(2.25) = 0.80$, $f(2.375) = 0.82$, $f(2.5) = 0.82$, then $\to 1$.

The law. Interpolating the contour $f = 0.68$ at each size:

R	κ^* measured	$\kappa^* = 2 + 1.83/R$	gap $(\kappa^*{-}2)R$	error
3	2.609	2.610	1.83	0.04%
4	2.459	2.458	1.84	0.06%
5	2.364	2.366	1.82	0.08%
6	2.302	2.305	1.81	0.13%
8	**2.228**	**2.229 (predicted first)**	1.83	0.04%

The $R = 8$ row is a genuine out-of-sample test: the constant-gap law was fit to $R \leq 6$ and used to predict $\kappa^*(8) = 2.229$ before the run, which returned 2.228. The alternative hypothesis — a fixed continuum shape ratio $\kappa^* \approx 2.3$ — is excluded by the same row.

Reading. The 0.68-contour (and, by the cliff's own drift toward $\kappa = 2$ with growing R, the whole transition) lives at a fixed *absolute* distance of order the lattice spacing between the ball surfaces. In continuum language: at fixed shape the transition collapses onto contact; the structure between the universal 1/2 of Sec. 3 and complete locking is an ultraviolet-scale feature of the vacuum, not a property of continuum geometry. The value $g = 1.83$ is scheme furniture (we note, and distrust, its proximity to $\ln 2\pi = 1.8379$); the scaling structure $g \propto \ell_{\mathrm{UV}}$ is the result.

5 Result 3: the vacuum's budget is collective, and transfers clear through it

Static decomposition. For one site A of the chain vacuum, the monogamous budget is $\tau(A{:}\text{rest}) = S_{1/2}(A)^2$ (pure cut). Summing pairwise contangles $\tau(A{:}B_i)$ over all mirror pairs B_i exhausts 11.3% of the budget at $m = 10^{-3}$ and 39.9% at $m = 0.1$; only the two nearest shells carry any pairwise negativity at all. The same decomposition attempted with mutual information sums to 27× the budget at criticality — the broadcast redundancy of a shared collective mode, and a quantitative caution against using I as a conservation currency.

Dynamical routing. Protocol: an external two-mode-squeezed pair (squeezing $s = 1.5$) is coupled to chain sites a, b ($|a - b| = 120$, $N = 240$, $m = 10^{-3}$) through beam splitters of transmissivity τ; beam splitters are passive and import entanglement without minting it. The contangle portfolio of site A:

τ	budget	$\tau(A{:}B)$	$\tau(A{:}\text{vac})$	$\tau(A{:}\text{anc})$	collective
0.000	2.792	0.000	1.815	0.000	0.978
0.146	4.360	0.000	0.014	0.001	4.346
0.309	5.591	0.000	0.000	0.000	**5.591 (100%)**
0.500	6.737	0.037	0.000	0.000	6.700
0.691	7.695	0.407	0.000	0.000	7.288
0.962	8.851	4.897	0.000	0.000	3.955
1.000	**9.0000**	**9.0000**	0.0000	0.0000	**0.0000**

The source account dies by $\tau \simeq 0.15$; the destination account is exactly zero until $\tau \simeq 0.5$; at $\tau = 0.31$ the budget is entirely multipartite; settlement closes the books to four decimals. Within this protocol, entanglement does not move from account to account: it dissolves into the collective sector and recondenses. Whether *any* Gaussian protocol admits a pairwise-hugging path — or whether clearing-through-the-collective is a theorem — we pose as an open question; the CKW structure of the static vacuum (above) suggests the latter.

6 Discussion

Three disclosures. *First*, on motivation: the contour value 0.68 in Sec. 4 was chosen because it equals the observed dark-energy fraction Ω_Λ; this work grew out of an emergent-gravity program in which the locked sector of vacuum correlation is conjectured to play the role of the cosmological constant [16]. Nothing in the present paper depends on that reading — the pure-cut value, the gap law, and the clearing-house routing are statements about free-field vacua — but the reader should know why 0.68 and not 0.5 or 0.9; the gap law holds with different g for any contour in the transition. *Second*, on novelty: the negativity-vanishing edge is Klco–Savage's [2, 3]; bound/inaccessible halo entanglement in lattice vacua is theirs as well; the contangle and its CKW property are Adesso–Illuminati's [9]. We believe the locked-*fraction* framing, the constant-gap scaling law with its out-of-sample test, the exact pure-cut 1/2 as a boundary value for this landscape, and the portfolio-resolved routing of a transfer protocol are new. Two adversarial literature audits (June 2026, $\sim$100 search-and-verify agents each) accompany the project's public records: the second specifically hunted prior art for the routing result of Sec. 5 and returned none — an absence-of-evidence verdict we report at its proper (low) confidence — while confirming, instructively, that a neighbouring claim we once entertained (lattice extraction of the modular temperature profile) is settled literature [12, 13, 14, 15], which is why no such claim appears in this paper. We welcome corrections. *Third*, on method: this work was produced in a human–AI collaboration in which the AI generated and audited derivations and code; we regard independent human verification — which the brevity of the scripts is designed to invite — as part of the publication process, not subsequent to it.

Open problems, in the order we would attack them: species universality of the gap law (fermions via the sine kernel; the photon field); the full continuum gap profile $f(g)$ at $L = 128$; the clearing-house theorem (does any protocol avoid the collective?); and the relation between the collective sector identified by CKW accounting and the undistillable sector identified by negativity death — two independent probes that, in all our runs, point at the same object.

7 Reproducibility

Every number above regenerates in seconds-to-minutes on a laptop from plain `numpy` scripts published at emergent-gravity.com: `d1_fstar.py` (FFT covariances, f measurements), `d3_constructions.py` and `d4_kstar_*.json` (contour scans and the $R = 8$ prediction), `n1b_ckw.json` (CKW saturation), `n2_withdrawal.py` and `n9_contangle_portfolio.json` (the transfer protocol and its portfolio), `n5_reserve.py` (negativity death / locked location). The $R = 8$ prediction precedes its measurement in the project's public git history.

References

[1] H. Casini and M. Huerta, "Entanglement entropy in free quantum field theory," J. Phys. A **42**, 504007 (2009), arXiv:0905.2562.

[2] N. Klco and M. J. Savage, "Geometric quantum information structure in quantum fields and their lattice simulation," Phys. Rev. D **103**, 065007 (2021), arXiv:2008.03647.

[3] N. Klco, D. H. Beck, and M. J. Savage, "Entanglement structures in quantum field theories: negativity cores and bound entanglement in the vacuum," Phys. Rev. A **107**, 012415 (2023), arXiv:2110.10736.

[4] G. Vidal and R. F. Werner, "Computable measure of entanglement," Phys. Rev. A **65**, 032314 (2002).

[5] M. B. Plenio, "Logarithmic negativity: a full entanglement monotone that is not convex," Phys. Rev. Lett. **95**, 090503 (2005).

[6] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, "Concentrating partial entanglement by local operations," Phys. Rev. A **53**, 2046 (1996).

[7] V. Coffman, J. Kundu, and W. K. Wootters, "Distributed entanglement," Phys. Rev. A **61**, 052306 (2000).

[8] T. J. Osborne and F. Verstraete, "General monogamy inequality for bipartite qubit entanglement," Phys. Rev. Lett. **96**, 220503 (2006).

[9] G. Adesso and F. Illuminati, "Continuous variable tangle, monogamy of entanglement, and entanglement frustration in multimode Gaussian states," New J. Phys. **8**, 15 (2006); "Entanglement in continuous-variable systems," J. Phys. A **40**, 7821 (2007), arXiv:quant-ph/0701221.

[10] G. Adesso and F. Illuminati, "Strong monogamy of bipartite and genuine multipartite entanglement," Phys. Rev. Lett. **99**, 150501 (2007), arXiv:quant-ph/0703277.

[11] I. Peschel, "Calculation of reduced density matrices from correlation functions," J. Phys. A **36**, L205 (2003).

[12] V. Eisler and I. Peschel, "Analytical results for the entanglement Hamiltonian of a free-fermion chain," J. Phys. A **50**, 284003 (2017), arXiv:1703.08126; "Properties of the entanglement Hamiltonian for finite free-fermion chains," J. Stat. Mech. 104001 (2018), arXiv:1805.00078.

[13] R. Arias, D. Blanco, H. Casini, and M. Huerta, "Local temperatures and local terms in modular Hamiltonians," Phys. Rev. D **95**, 065005 (2017), arXiv:1611.08517.

[14] V. Eisler, E. Tonni, and I. Peschel, "On the continuum limit of the entanglement Hamiltonian," J. Stat. Mech. 073101 (2019), arXiv:1902.04474.

[15] G. Di Giulio and E. Tonni, "On entanglement Hamiltonians of an interval in massless harmonic chains," J. Stat. Mech. 033102 (2020), arXiv:1911.07188.

[16] The Emergent Gravity Project, "Monogamy Gravity: the Ledger," emergent-gravity.com (2026), Folios I–XIII.